Presentation for Team-Meeting of Group of Risk and Safety, ETH Zürich

Global seismic reliability analysis for building structures

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Outline

- Why global reliability analysis?
- Global load-carrying capacity limit state function
- Stochastic moments of global performance function
- Methodologies for global seismic reliability analysis
- Applications of the methodology to RC framed structures
- Outlook

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Why global reliability analysis?

- **Failure mode approach (FMA) for classical system reliability**

  - **The constitutive relations of materials are assumed to be perfect rigid-plastic**
  - **It is hardly to identify the significant failure modes and determine their corresponding failure mode equations**
  - **The correlation between failure modes is another important problem not easy to deal with**
  - **The overall failure probability of structural systems cannot be evaluated accurately even though the dominant failure modes and limit state equations are known a prior**

Why global reliability analysis?

❑ Global reliability theory of structures

- **Global limit states based on nonlinear structural analysis techniques**
- **The integrated nonlinear analysis methods of structural systems considering real constitutive relations of materials are utilized to search for the dominant modes of failure**
- **The statistics of structural global load-carrying capacity are obtained by Monte Carlo simulations**
- **The probability density function (PDF) of the global load-carrying capacity is fitted by its first few moments**
- **The classic structural component reliability theory can be applied in the global limit state equation**

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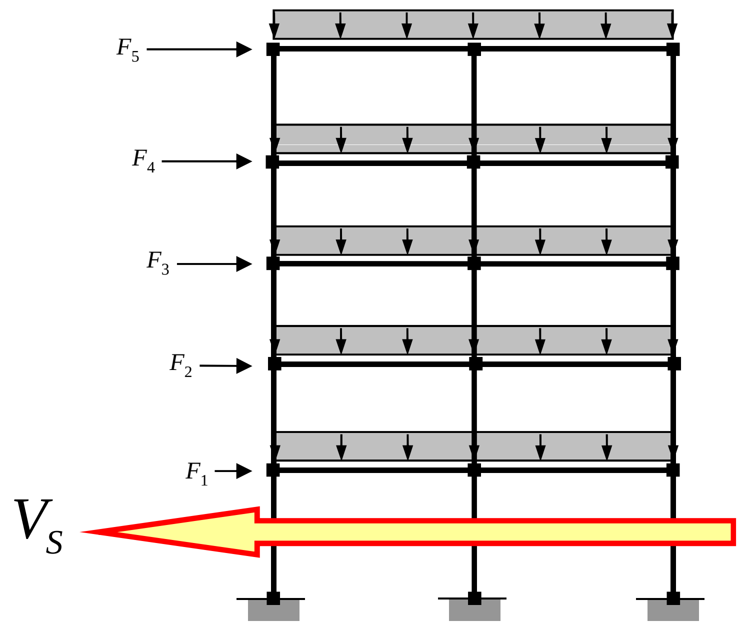
Global load-carrying capacity LSF

❑ Global LSF Based on Limit Base Shear

$$g(V_S, F_E) = V_S - F_E$$

V_S = limit base shear of structures

F_E = total horizontal seismic action of structures in the base



Global load-carrying capacity LSF

❑ Limit Base Shear of Structures

- **Pushover analysis**
- **Nonlinearity and randomness**

$$F_{V_S}(v) = \Phi\left(\frac{\ln v - \lambda_{V_S}}{\zeta_{V_S}}\right) \qquad \lambda_{V_S} = \ln\left(\frac{\mu_{V_S}}{\sqrt{1+\delta_{V_S}^2}}\right)$$

$$\zeta_{V_S} = \sqrt{\ln\left(1+\delta_{V_S}^2\right)}$$

Global load-carrying capacity LSF

❑ Equivalent Static Random Seismic Action

- **Equivalent SDOF**

$$F_E = \alpha G D = \frac{A_m}{g}\beta(T,\xi) G D$$

- **Probability model of equivalent static random seismic action**

$$F_{F_E}(f \mid I = J) = \exp\{-\exp[-\alpha(f-u)]\}$$

$$\alpha = \frac{\pi}{\sqrt{6}\mu_{F_J} V_{F_J}}$$

$$u = \mu_{F_J}(1 - 0.45 V_{F_J})$$

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Stochastic moments of global LSF

Point Estimation Methods Based on Nataf Transformation

- **The forward Nataf transformation**

$$T_N : \mathbf{u} = \mathbf{L}_0^{-1}\mathbf{\Phi}^{-1}[\mathbf{F}_{\mathbf{X}}(\mathbf{x})]$$

- **The inverse Nataf transformation**

$$T_N^{-1} : \mathbf{x} = \mathbf{F}_{\mathbf{X}}^{-1}[\mathbf{\Phi}(\mathbf{L}_0\mathbf{u})]$$

- **The first few moments of random function**

$$\mu_Z = \int g(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \int g\left[T_N^{-1}(\mathbf{u})\right]\varphi_n(\mathbf{u}) d\mathbf{u}$$

$$M_{kZ} = \int \left[g(\mathbf{x}) - \mu_Z\right]^k f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

$$= \int \left\{g\left[T_N^{-1}(\mathbf{u})\right] - \mu_Z\right\}^k \varphi_n(\mathbf{u}) d\mathbf{u} \quad \text{for } k \geq 2$$

Stochastic moments of global LSF

❑ PEM for Single Variable Function

- **Gauss-Hermite numerical quadrature in standard normal space**

$$\mu_Z \approx \sum_{j=1}^{m} P_j g\left\{F_X^{-1}[\Phi(u_j)]\right\}$$

$$\sigma_Z^2 \approx \sum_{j=1}^{m} P_j \left[g\left\{F_X^{-1}[\Phi(u_j)]\right\} - \mu_Z \right]^2$$

$$\alpha_{rZ}\sigma_Z^r \approx \sum_{j=1}^{m} P_j \left[g\left\{F_X^{-1}[\Phi(u_j)]\right\} - \mu_Z \right]^r \quad r = 3, 4$$

Stochastic moments of global LSF

❑ PEM for Multiple Variable Function

- **The function is approximated by a non-product function**

$$g(\mathbf{X}) \approx g'(\mathbf{X}) = \sum_{i=1}^{n} (G_i - G_{\boldsymbol{\mu}}) + G_{\boldsymbol{\mu}}$$

$$G_{\boldsymbol{\mu}} = g(\boldsymbol{\mu}) = g(\mu_1, \cdots, \mu_i, \cdots, \mu_n)$$

$$\begin{aligned} G_i &= g\left[T_N^{-1}(\mathbf{u}_i)\right] = G(\mathbf{u}_i) \\ &= G(u_{\mu 1}, u_{\mu 2}, \cdots, u_{\mu i-1}, u_i, u_{\mu i+1}, \cdots, u_{\mu n}) \end{aligned}$$

Stochastic moments of global LSF

❑ PEM for Multiple Variable Function

- **The function is approximated by a non-product function**

$$\mu_Z \approx \sum_{i=1}^{n} (\mu_i - G_{\boldsymbol{\mu}}) + G_{\boldsymbol{\mu}}$$

$$\sigma_Z^2 \approx \sum_{i=1}^{n} \sigma_i^2$$

$$\alpha_{3Z}\sigma_Z^3 \approx \sum_{i=1}^{n} \alpha_{3i}\sigma_i^3$$

$$\alpha_{4Z}\sigma_Z^4 \approx \sum_{i=1}^{n} \alpha_{4i}\sigma_i^4 + 6\sum_{i=1}^{n-1}\sum_{j>i}^{n} \sigma_i^2\sigma_j^2$$

Stochastic moments of global LSF

❑ Random POA by PEM

- **Build the finite element model of structure**
- **Determine the probability distribution types and their distribution parameters of basic random variables that influence the limit base shear of structures;**
- **Generate structural samples by sampling of the basic random variables**
- **Conduct pushover analysis for each structural sample to derive its base shear-top displacement curve, from which the limit base shear is obtained**
- **Compute the first two statistical moments of limit base shear**

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Global seismic reliability analysis methods

- **First Order Reliability Method (FORM)**

$$p_f = P[g(\boldsymbol{X}) \le 0] = \int_{g(\boldsymbol{x})\le 0} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} = \int_{G(\boldsymbol{u})\le 0} \varphi_n(\boldsymbol{u})d\boldsymbol{u}$$

$$G(\boldsymbol{u}) \approx G(\boldsymbol{u}^*) + \nabla G(\boldsymbol{u}^*)^T(\boldsymbol{u}-\boldsymbol{u}^*) = \nabla G(\boldsymbol{u}^*)^T(\boldsymbol{u}-\boldsymbol{u}^*)$$

$$\nabla G(\boldsymbol{u}_i) = \left[\boldsymbol{J}_{\boldsymbol{y},\boldsymbol{u}}(\boldsymbol{y}_i, \boldsymbol{u}_i)\right]^T \nabla g(\boldsymbol{y}_i)$$

$$\boldsymbol{\alpha}^* = -\nabla G(\boldsymbol{u}^*) / \| \nabla G(\boldsymbol{u}^*) \|$$

$$\boldsymbol{u}_{i+1} = \left(\frac{G(\boldsymbol{u}_i)}{\left\|\nabla G(\boldsymbol{u}_i)\right\|} + \boldsymbol{\alpha}_i^T \boldsymbol{u}_i \right) \boldsymbol{\alpha}_i$$ HLRF algorithm

$$\beta_{\mathrm{HL}} = \boldsymbol{\alpha}^{*T} \boldsymbol{u}^* \qquad p_f \approx \Phi\left(-\beta_{\mathrm{HL}}\right)$$

Global seismic reliability analysis methods

❑ Moment Methods (MM)

- **Second Moment Formulation**

$$\beta_{2\mathrm{M}} = \frac{\mu_{\mathrm{Z}}}{\sigma_{\mathrm{Z}}}$$

$$p_{f\,2\mathrm{M}} = \Phi(-\beta_{2\mathrm{M}})$$

Global seismic reliability analysis methods

❑ Moment Methods (MM)

- **Third Moment Formulation**

$$Z_u = \frac{Z - \mu_Z}{\sigma_Z}$$

$$\beta_{3M} = \frac{-\text{sign}(\alpha_{3Z})}{\sqrt{\ln(A)}} \ln\left[\sqrt{A}\left(1 + \frac{\beta_{2M}}{u_b}\right)\right]$$

$$p_{f3M} = \Phi(-\beta_{3M})$$

$$A = 1 + \frac{1}{u_b^2}$$

$$u_b = (a+b)^{1/3} + (a-b)^{1/3} - \frac{1}{\alpha_{3Z}}$$

$$a = \frac{1}{\alpha_{3Z}}\left(\frac{1}{\alpha_{3Z}^2} + \frac{1}{2}\right),$$

$$b = -\frac{1}{2\alpha_{3Z}^2}\sqrt{\alpha_{3Z}^2 + 4}$$

Global seismic reliability analysis methods

❑ Moment Methods (MM)

- **Forth Moment Formulation 1**

$$\beta_{4M\_1} = \frac{3(\alpha_{4Z} - 1)\beta_{2M} - \alpha_{3Z}(\beta_{2M}^2 - 1)}{\sqrt{(9\alpha_{4Z} - 5\alpha_{3Z}^2 - 9)(\alpha_{4Z} - 1)}}$$

$$p_{f\,4M\_1} = \Phi(-\beta_{4M\_1})$$

Global seismic reliability analysis methods

❑ Moment Methods (MM)

- **Forth Moment Formulation 2**

$$F(z_u) = \Phi(z_u) - \varphi(z_u)\left[\frac{1}{6}\alpha_{3Z} H_2(z_u) + \cdots + \frac{1}{24}(\alpha_{4Z} - 3) H_3(z_u) + \frac{1}{72}\alpha_{3Z}^2 H_5(z_u)\right]$$

$$H_2(x) = x^2 - 1, H_3(x) = x^3 - 3x, H_5(x) = x^5 - 10x^3 + 15x$$

$$p_{f\,4M\_2} = \Phi(-\beta_{2M}) - \varphi(\beta_{2M})\left[\frac{1}{6}\alpha_{3Z} H_2(-\beta_{2M}) + \frac{1}{24}(\alpha_{4Z} - 3) H_3(-\beta_{2M}) + \frac{1}{72}\alpha_{3Z}^2 H_5(-\beta_{2M})\right]$$

$$\beta_{4M\_2} = \Phi^{-1}(p_{f\,4M\_2})$$

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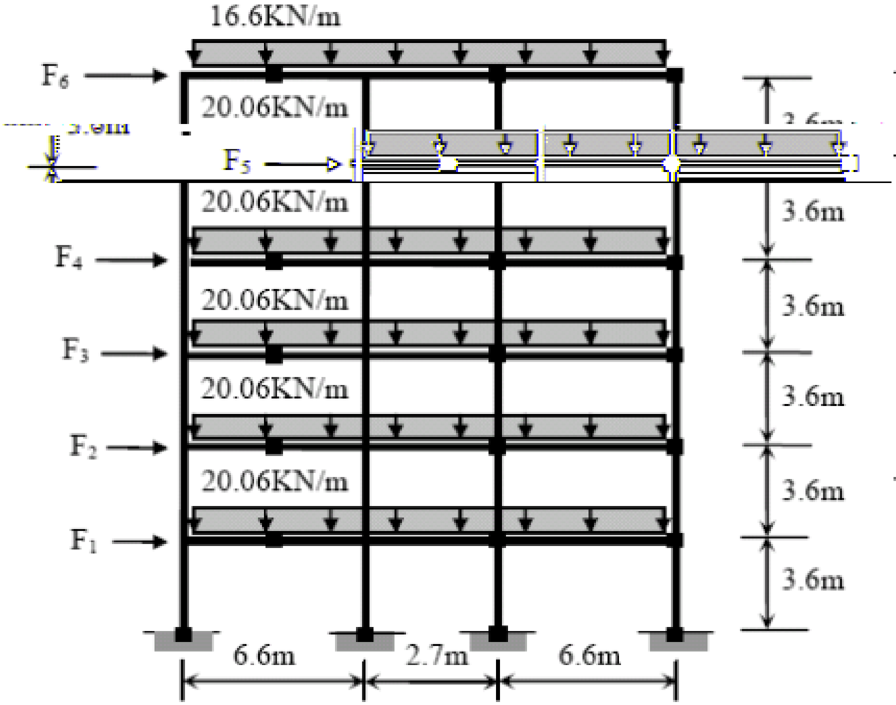
Applications: RC Framed Structures

❑ Structural Modeling



Members	Height (mm)	Width (mm)	Concrete Grade	Rebar Grade	Hoops Grade
Interior Columns	500	500	C30	HRB335	HPB235
Exterior Columns	500	500	C30	HRB335	HPB235
Main Beams	600	300	C30	HRB335	HPB235
Side Beams	500	200	C30	HRB335	HPB235

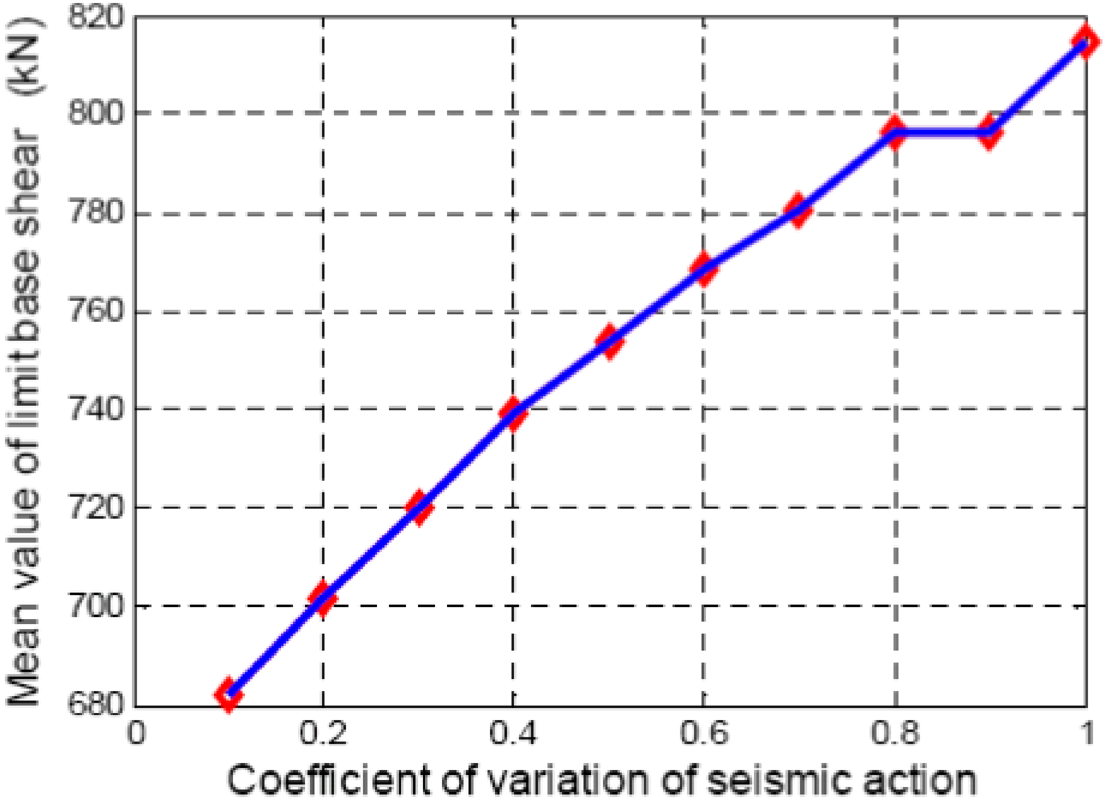
Applications: RC Framed Structures

❑ Probability Modeling

RVs	Mean value	Std	COV	Types	Corr. Coef.
f_c (N/mm^2)	14.3	2.86	0.2	Log-normal	0.3
f_y (N/mm^2)	363	72.6	0.2		
E (N/mm^2)	2×10^5	0.4×10^5	0.2		
α	0.05	0.001	0.2	Normal	
F_1 (KN)	33.513	3.351~33.513	0.1~1.0	Type-I largest	0.0~0.9
F_2 (KN)	58.379	5.838~58.379	0.1~1.0		
F_3 (KN)	80.345	8.035~80.345	0.1~1.0		
F_4 (KN)	98.292	9.829~98.292	0.1~1.0		
F_5 (KN)	111.170	11.117~111.170	0.1~1.0		
F_6 (KN)	91.721	9.172~91.721	0.1~1.0		

Applications: RC Framed Structures

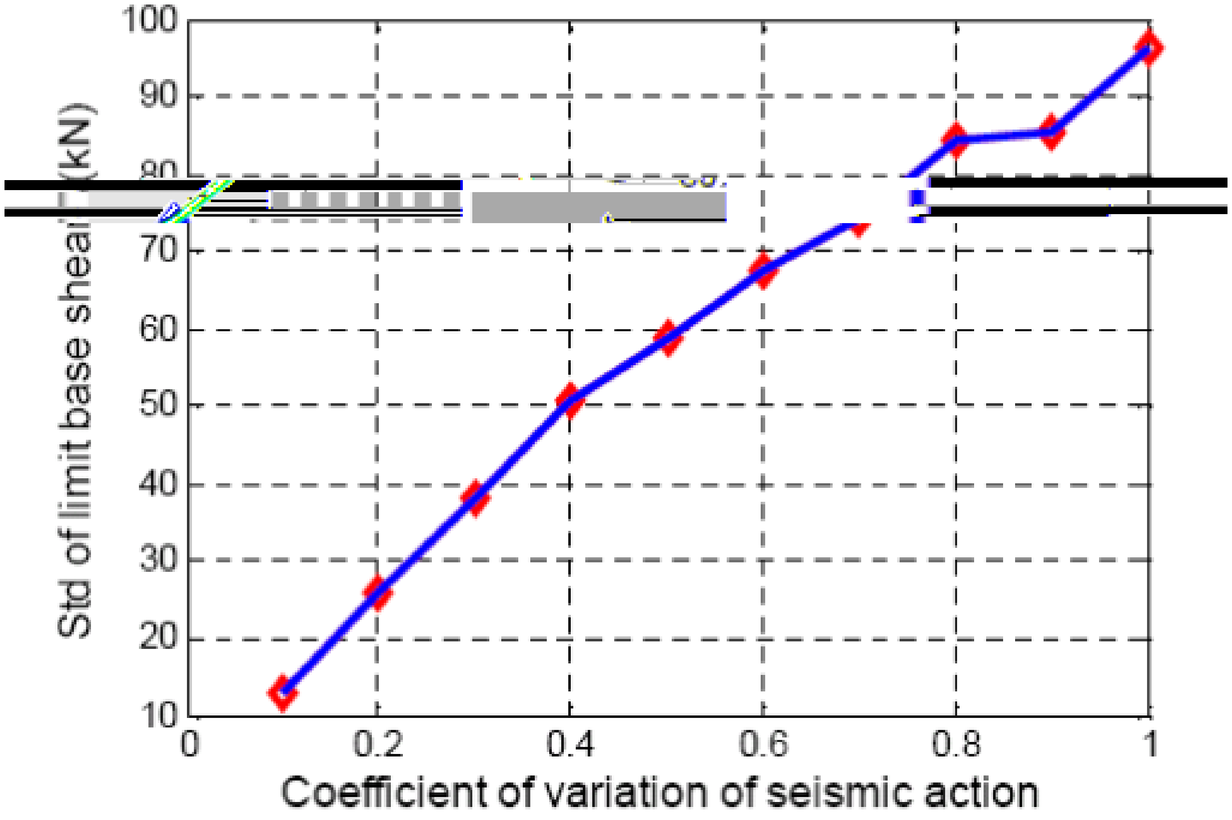
❑ Probability Analysis of Limit Base Shear



Variations of the mean with COV of total horizontal seismic action

Applications: RC Framed Structures

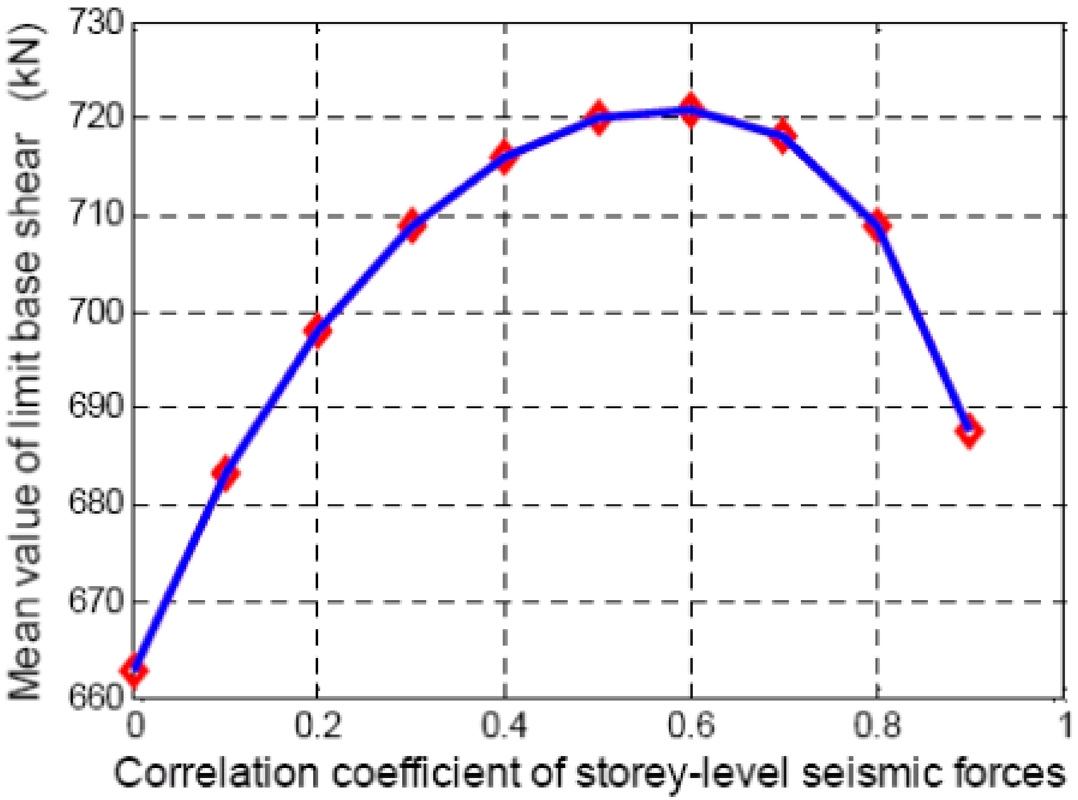
❑ Probability Analysis of Limit Base Shear



Variations of the Std with COV of total horizontal seismic action

Applications: RC Framed Structures

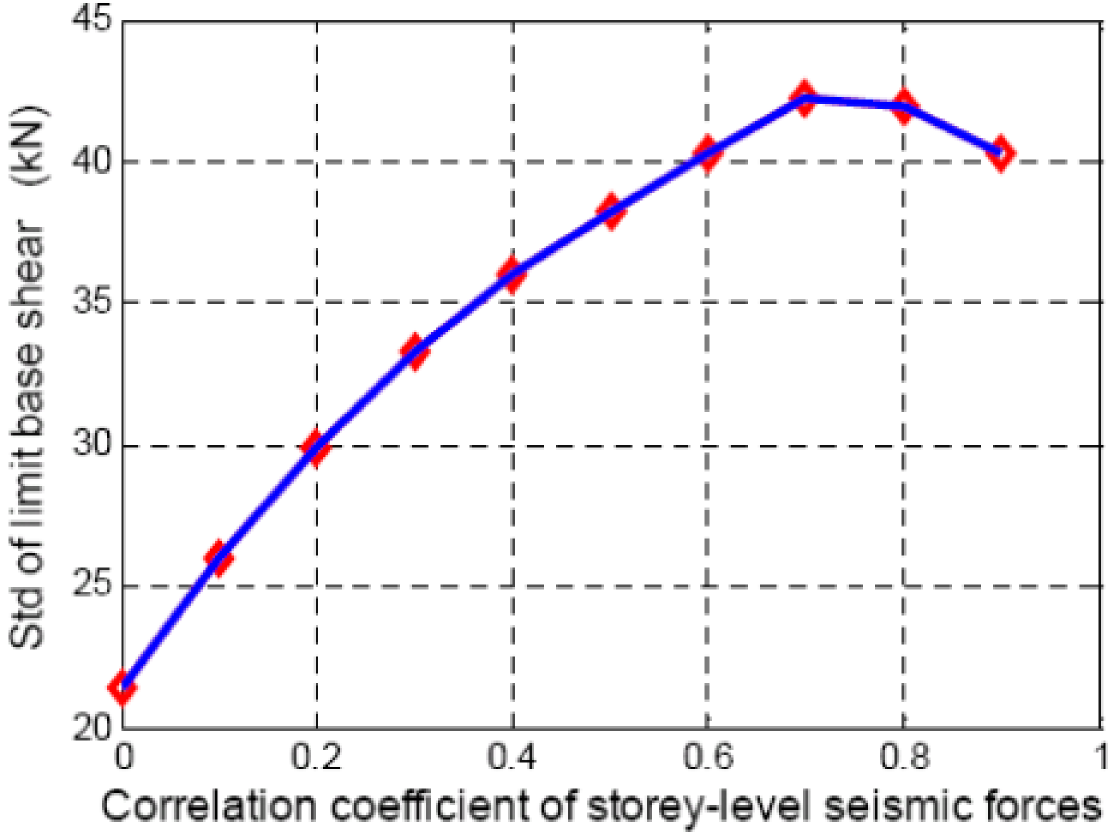
❑ Probability Analysis of Limit Base Shear



Variations of the mean with correlation coefficient of storey-level seismic forces

Applications: RC Framed Structures

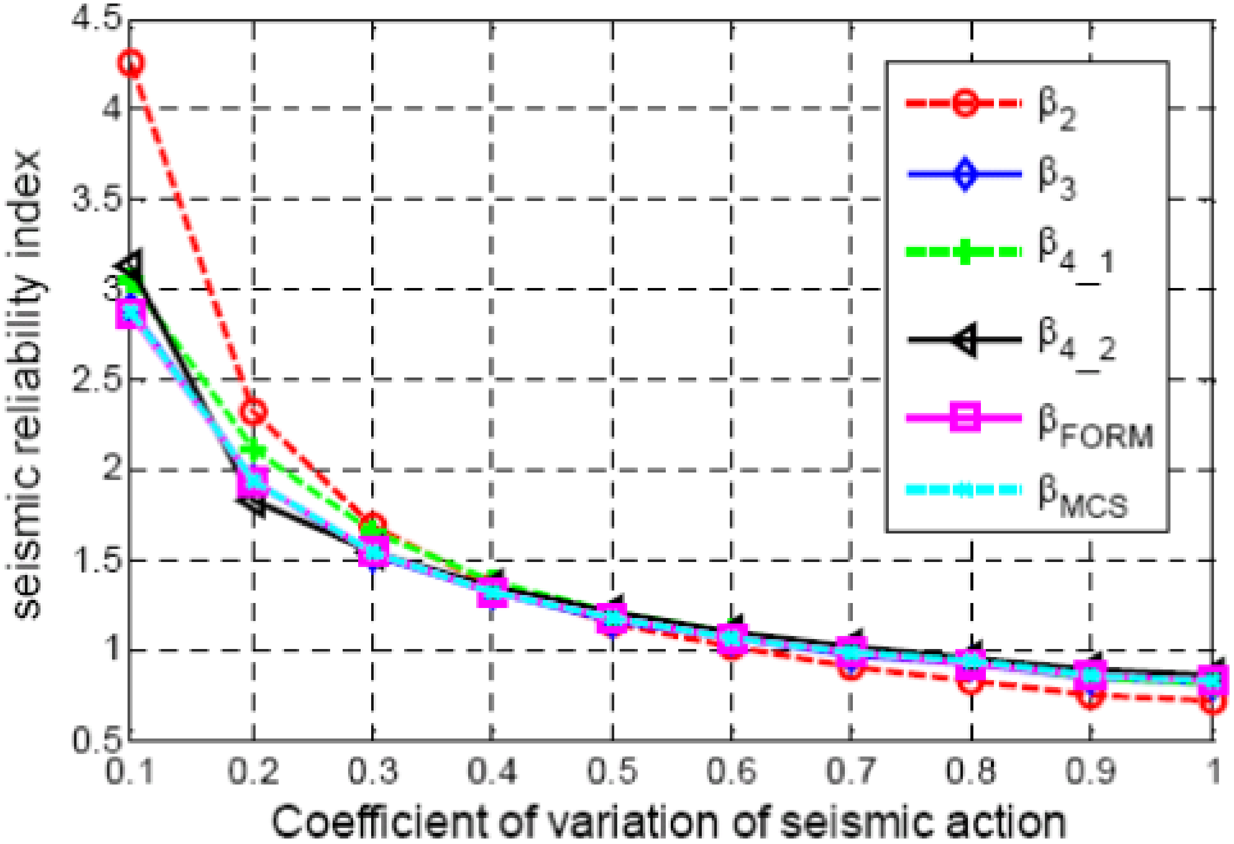
- ❑ **Probability Analysis of Limit Base Shear**



Variations of the Std with correlation coefficient of storey-level seismic forces

Global load-carrying capacity LSF

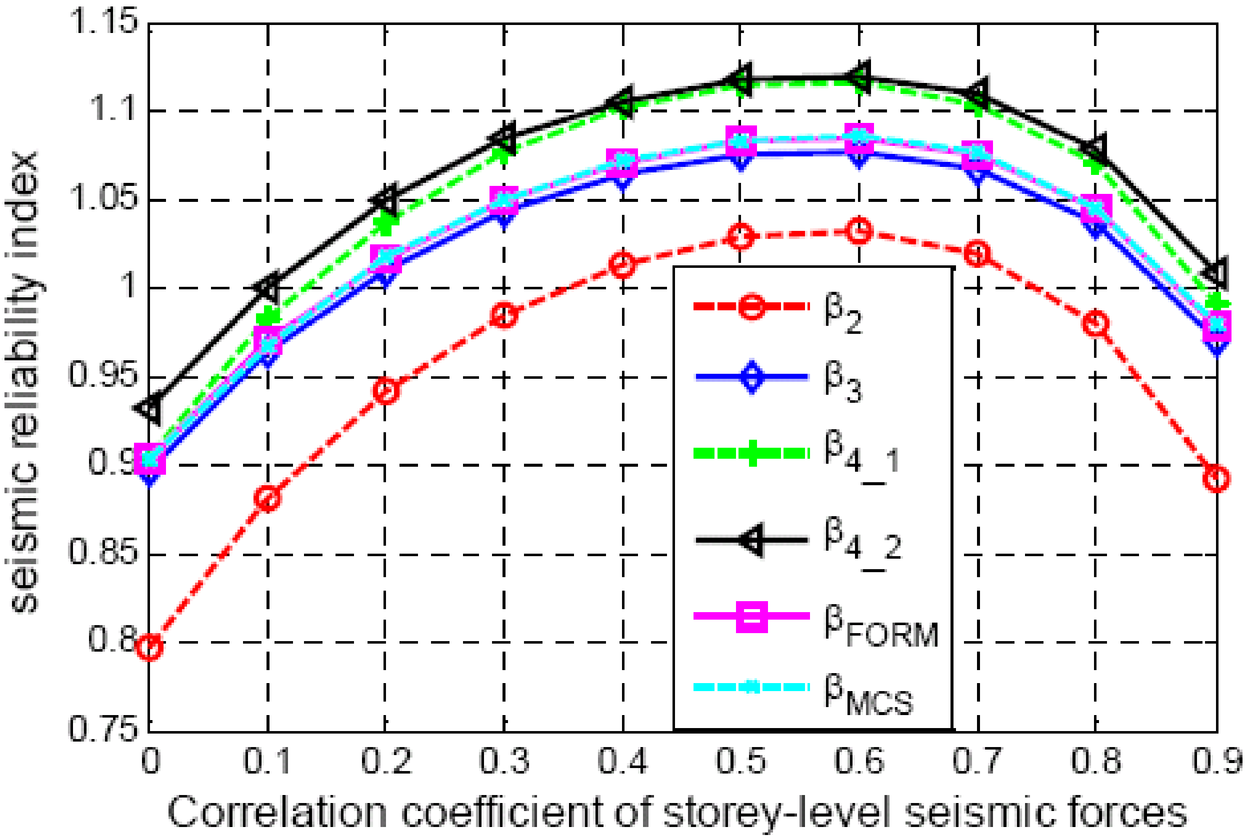
❑ Global Seismic Reliability Index



Variations of GSR with COV of seismic action

Global load-carrying capacity LSF

❑ Global Seismic Reliability Index



Variations of GSR with correlation coefficient of storey-level seismic forces

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Outlook

- **Progressive Collapse Analysis (PCA)**

$$P(\text{Collape}) = \sum_{H}\sum_{D} P(\text{Collape} \mid D) P(D \mid H) P(H)$$

$$P(\text{Collapse} \mid \text{Scenario}) = \sum_{D} P(\text{Collapse} \mid D) P(D \mid \text{Scenario})$$

$$P(\text{Collapse} \mid D)$$

How to compute the Conditional Collapse Probability (CCP)?

By using global reliability method !

Thank you for your attention!

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