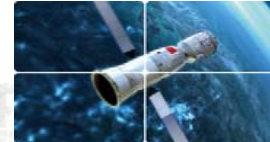




哈爾濱工業大學  
HARBIN INSTITUTE OF TECHNOLOGY



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*Presentation for Team-Meeting of Group of Risk and Safety, ETH Zürich*

# Risk Based Minimum Life-Cycle Cost Design of Aseismic Structures

**Dagang LU**

**Prof. Dr. of Harbin Institute of Technology**

**Academic Guest of Group of Risk and Safety, ETH Zürich**

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# Outline

- ❑ Introduction
- ❑ Risk-benefit-cost criteria for seismic design of structures
- ❑ Minimum life-cycle cost design methodology of aseismic structures
- ❑ Probabilistic seismic risk analysis of structures
- ❑ Applications of the methodology to steel frame buildings
- ❑ Conclusions

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# Introduction

## □ The current seismic design methodology

- Level II probability-based limit states design
- *Load and Resistance Factor Design* (LRFD) formulation
- Based on notional failure probability of structural components by calibrating level I structural design codes
- Cannot explicitly consider the consequences of earthquake events in terms of seismic loss, life-cycle cost, or even fatalities rate

# Introduction

## □ From *reliability-based design* to *risk-based design*

- Low-probability and high-consequence natural disasters
- Risk is most meaningful and useful than reliability when expressed in terms of potential economic losses and/or human sufferings
- A new paradigm of *risk-based design*
- Quantitative risk analysis tools
- Risk-benefit-cost criterion for seismic design of structures

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## Risk-benefit-cost criteria for seismic design of structures

### □ Existing models of risk-benefit-cost criteria

$$E[C_T(\mathbf{d})] = C_I(\mathbf{d}) + E[C_D(\mathbf{d})] \quad \text{Liu \& Neghabat (1972)}$$

$$C_{tot} = C_b + C_m + \sum P_f C_f \quad \text{ISO 2394 (1998)}$$

$$E[C(t, \mathbf{x})] = C_0 + \frac{v}{\lambda} (1 - e^{-\lambda t}) \sum_i^k C_i P_i + \frac{(1 - e^{-\lambda t})}{\lambda} C_m$$

Wen and Kang (2001a,b)

$$E[C_T(p_f)] = C_I(p_f) + C_m(p_f) + E[C_D(p_f)]$$

Ang & De Leon (1997)

## Risk-benefit-cost criteria for seismic design of structures

### □ A new model of risk-informed decision-making

*Fortification intensity*  $I_d$  instead of the *target reliability*

$$C_{tot} = C_I[\mathbf{d}(I_d)] + \frac{V}{\lambda} (1 - e^{-\lambda t}) L[\mathbf{d}(I_d)]$$

*Minimum cost function* instead of the *initial cost function*

$$E[C(t, I_d)] = w_1 C_{\min}[\mathbf{d}(I_d)] + w_2 \frac{V}{\lambda} (1 - e^{-\lambda t}) L[\mathbf{d}(I_d)]$$

$$E[C(t, I_d)] = w_1 C_{\min}(I_d) + w_2 \frac{V}{\lambda} (1 - e^{-\lambda t}) L(I_d)$$



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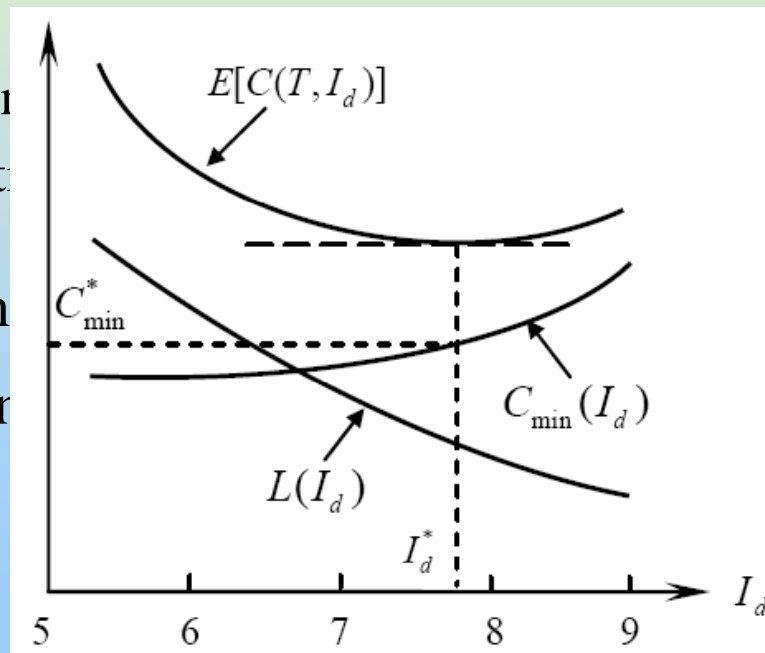
## Minimum life-cycle cost design methodology of aseismic structures

### □ Two-stage optimization methodology for minimum life-cycle cost design

**Stage 1:** Decision-making for the optimal fortification intensity of aseismic structures considering expected life cycle cost

$$E[C(T, I_d)] = C_{\min}(I_d) + \frac{V}{\lambda}(1 - e^{-\lambda T})L(I_d) \rightarrow \min$$

**Stage 2:** Min  
fort  
Fin  
min  
s.t.



der the optimal

Decision-Making for the  
Optimal Fortification Intensity

## Minimum life-cycle cost design methodology of aseismic structures

### □ Minimum initial cost design of aseismic structures

For a given  $I_d$

Find  $\mathbf{d}(I_d)$

min  $C[\mathbf{d}(I_d)]$

s.t. requirements of design codes

Polak-Ribiere conjugate gradient direction algorithm

- search direction

$$\mathbf{r}^{(j)} = -\nabla Q(\mathbf{d}^{(j)}) + \theta_{j-1} \mathbf{r}^{(j-1)}$$

- conjugate direction coefficient

$$\theta_{j-1} = \frac{\left[ \nabla Q(\mathbf{d}^{(j)}) - \nabla Q(\mathbf{d}^{(j-1)}) \right]^T \nabla Q(\mathbf{d}^{(j)})}{\left\| \nabla Q(\mathbf{d}^{(j-1)}) \right\|^2}$$

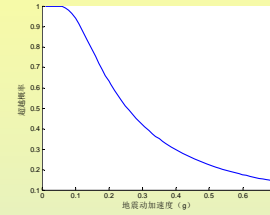
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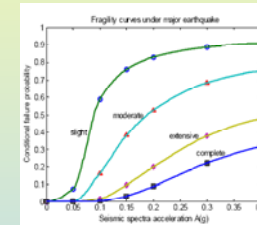
# Probabilistic seismic risk analysis of structures

## General Framework for PSRA

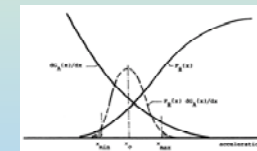
**Probabilistic Seismic Hazard Analysis (PSHA)**



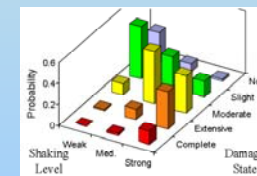
**Probabilistic Seismic Fragility Analysis (PSFA)**



**Probabilistic Seismic Safety Analysis (PSSA)**



**Probabilistic Seismic Damage Analysis (PSDA)**



**Probabilistic Seismic Loss Analysis (PSLA)**

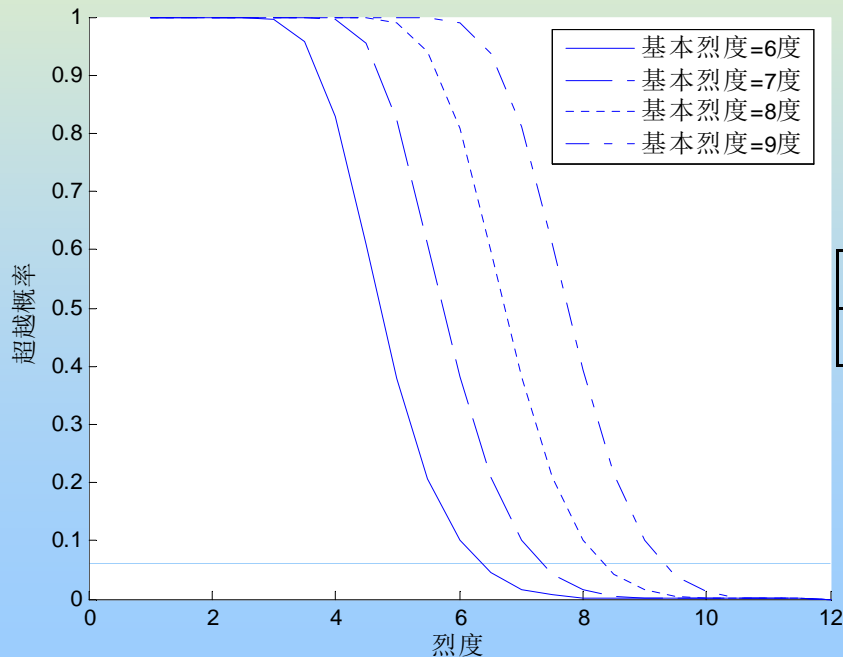
- casualties (Deaths)
- direct damage (Dollars)
- loss of use (Downtime)

# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Hazard Analysis of Sites

- PSHA for general sites in mainland of China

$$F_I(i) = \exp\left[-\left(\frac{\omega - i}{\omega - \varepsilon}\right)^k\right] \quad \text{type III extreme value distribution}$$



Basic intensity $I_0$	6	7	8	9
$k$	9.7932	8.3339	6.8713	5.4028

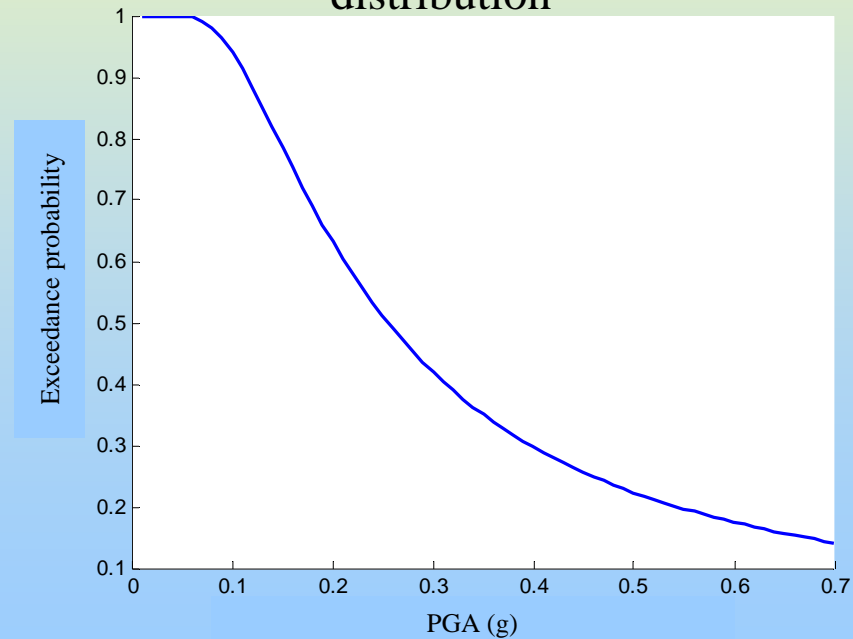
# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Hazard Analysis of Sites

- PSHA for specific sites

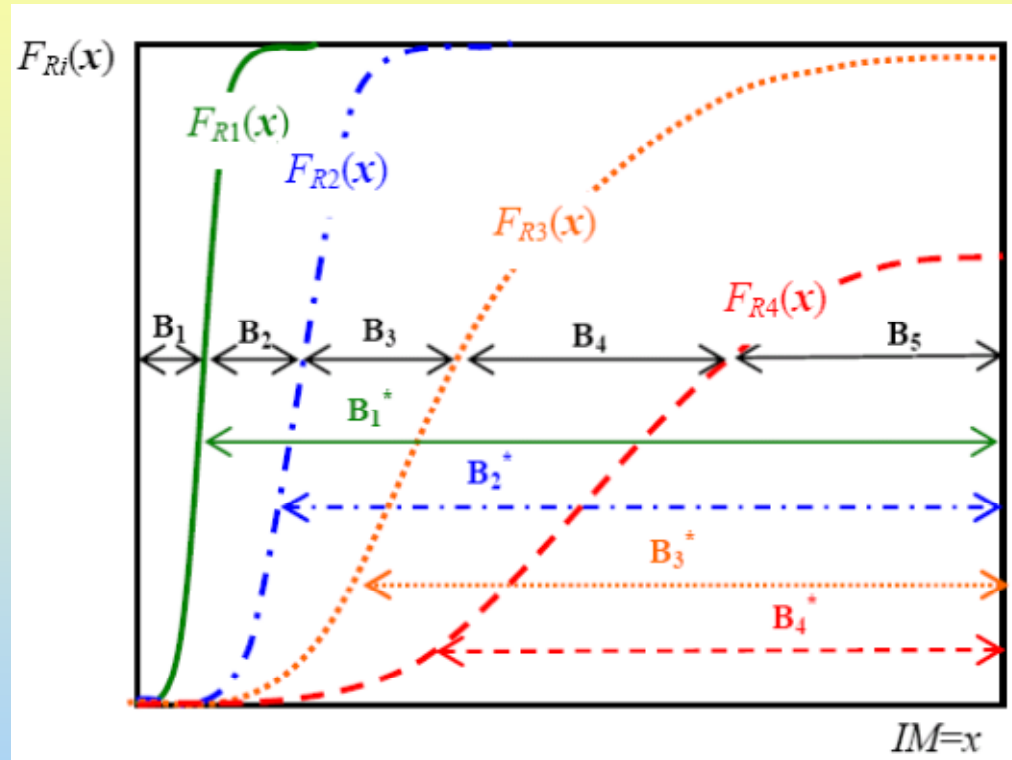
$$H_A(a) = 1 - \exp[-(x/k_0)]^{-k} \approx k_0 x^{-k}$$

CCDF of Type II extreme value distribution



# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Fragility Analysis of Structures



$$F_R(x) = P[D \geq C \mid IM = x] = F_{IM,C}(x)$$

$$F_{R_i}(x) = P_f[B_i^* \mid x] = P[D \geq C_i \mid IM = x]$$

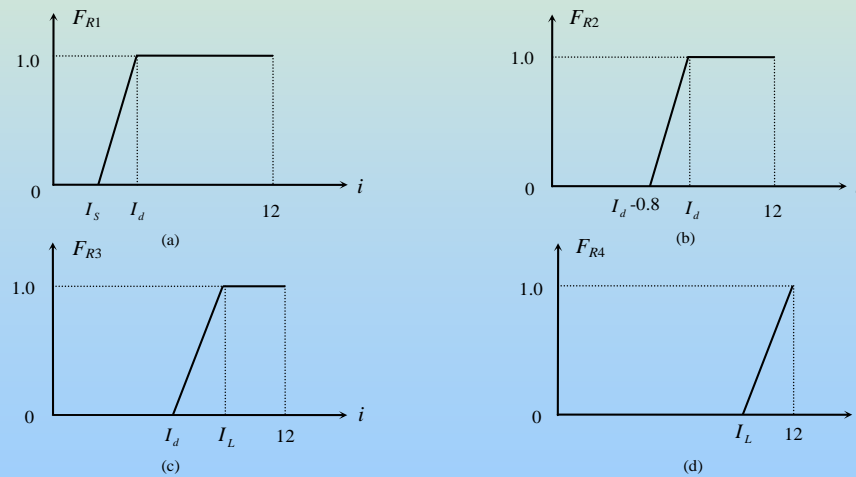


# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Fragility Analysis of Structures

- PSFA for general structures

Earthquake levels	Minor earthquake ( $I_s$ )	Moderate earthquake ( $I_M$ )	Major earthquake ( $I_L$ )
Exceedance probability in 50 years	0.632	0.10	0.02 to 0.03
Relationships with the basic intensity $I_0$	$I_s = I_0 - 1.55$	$I_m = I_0$	$I_L \approx I_0 + 1$
Performance objectives	Do not be damaged	Can be repaired	Do not collapse



Simplified fragility curves for four limit states

# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Fragility Analysis of Structures

- PSFA for specific structures

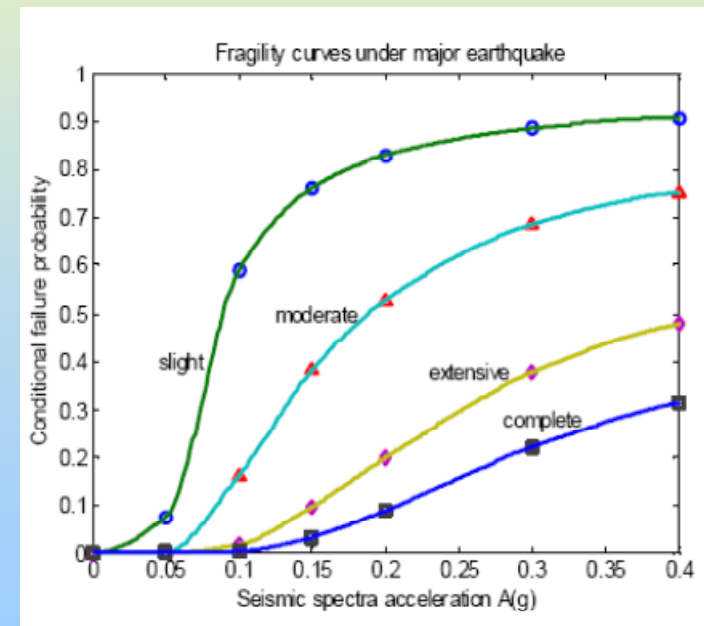
$$P_f[B_j^*, \mathbf{d}(I_d) | I_k] = P[\Delta(I_d, I_k) > \Delta_j]$$

$$F_R(x) = \Phi[\ln(x/m_R)/\beta_R]$$

Performance and damage levels in terms of inter-storey drift ratio

Performance level	Damage State	Drift ratio (%)
I	B <sub>1</sub> : None	$\Delta < 0.2$
II	B <sub>2</sub> : Slight	$0.2 < \Delta < 0.4$
III	B <sub>3</sub> : Moderate	$0.4 < \Delta < 0.8$
IV	B <sub>4</sub> : Severe	$0.8 < \Delta < 2.0$
V	B <sub>5</sub> : Collapse	$\Delta > 2.0$

## Limit State Fragility Curves



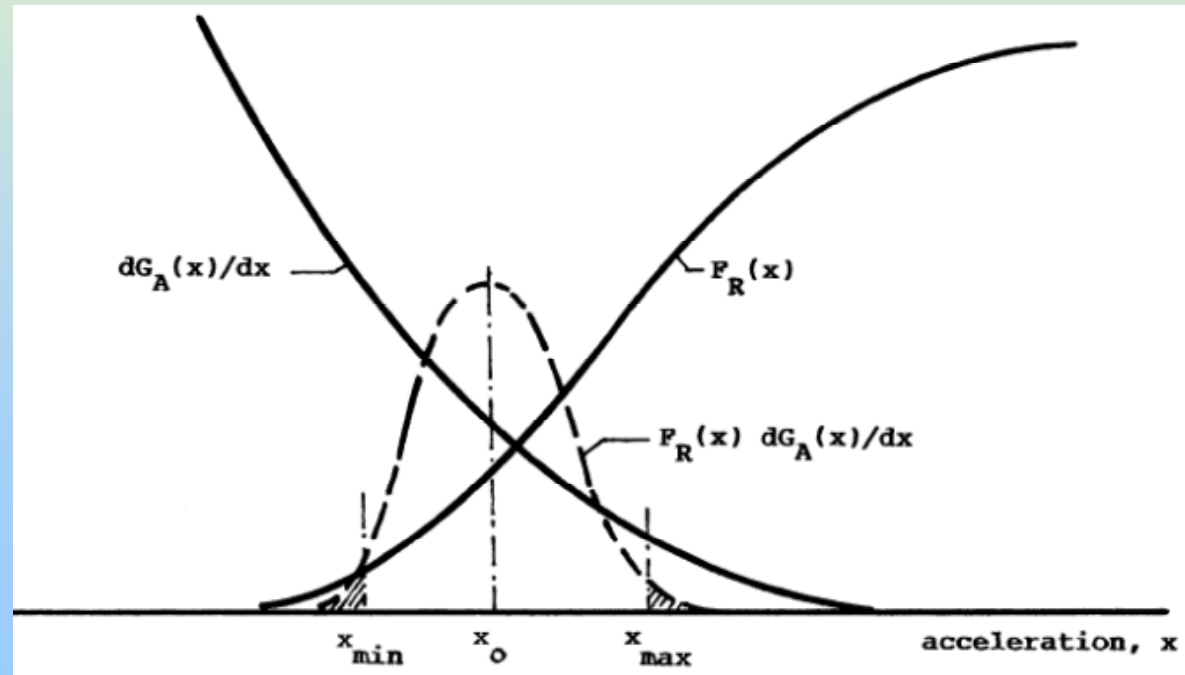
# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Safety Analysis of Structures

### Limit State Probability of PSSA

$$P_{LS} = P[D \geq C] = \sum_x P[D \geq C | IM = x] P[IM = x]$$

$$P_{LS} = \int_0^{\infty} F_R(x) |dH(x)| = \int_0^{\infty} F_{IM,C}(x) |dH(x)|$$



# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Safety Analysis of Structures

- Numerical integration method of PSSA

$$P_f[B_j^*, \mathbf{d}(I_d)] = \sum_{I_k} P_f[B_j^*, \mathbf{d}(I_d) | I_k] \cdot P(I_k)$$

$$P(I_k = 6.0) = F_I(6.25),$$

$$P(I_k = 6.5) = F_I(6.75) - F_I(6.25),$$

$$P(I_k = 7.0) = F_I(7.25) - F_I(6.75),$$

$$P(I_k = 7.5) = F_I(7.75) - F_I(7.25),$$

$$P(I_k = 8.0) = F_I(8.25) - F_I(7.75),$$

$$P(I_k = 8.5) = F_I(8.75) - F_I(8.25),$$

$$P(I_k = 9.0) = 1 - F_I(8.75)$$

# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Safety Analysis of Structures

- Analytical approximate method of PSSA

$$P_{LS_j} = P_f [B_j^*, \mathbf{d}(I_d)] = \int_0^\infty F_{R_j}(x) d | H_{S_a}(x) |$$

$$P_f [B_j^*, \mathbf{d}(I_d)] = H_A(m_R) \exp[(k \beta_R)^2 / 2]$$

(Cornell, 1996)

$$P_f [B_j^*, \mathbf{d}(I_d)] = H(s_a^{m_c}) \exp \left[ \frac{1}{2} \frac{k^2}{b^2} (\beta_{D|S_a}^2 + \beta_C^2) \right]$$

(Cornell, 2002)

# Probabilistic seismic risk analysis of structures

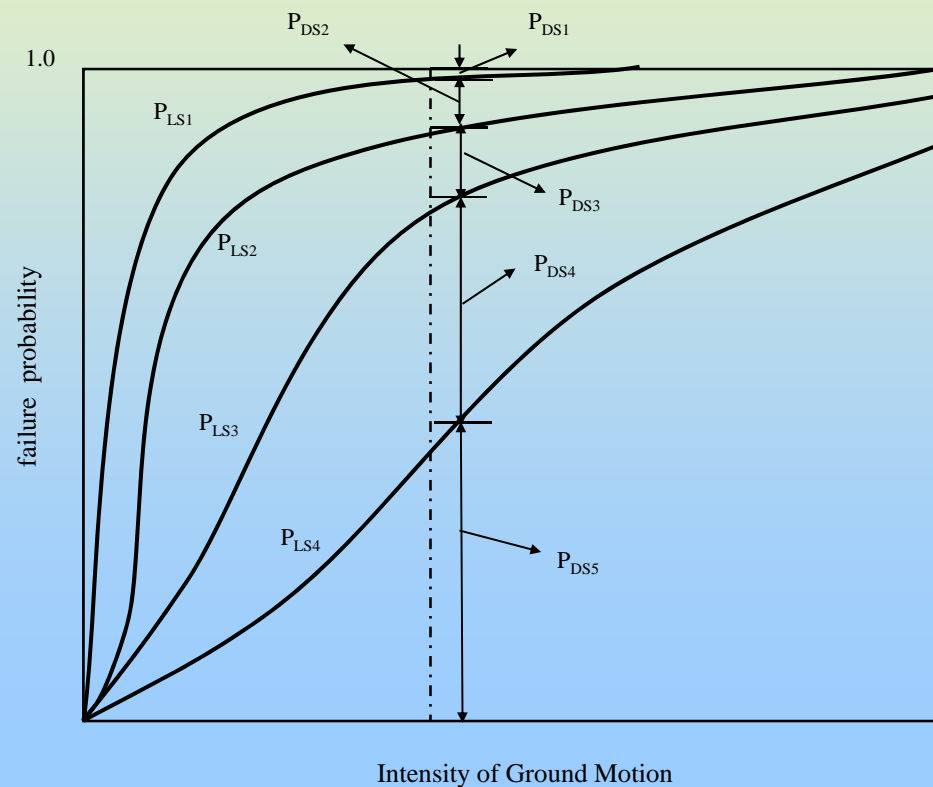
## □ Probabilistic Seismic Damage Analysis of Structures

### Damage State Probabilities of PSDA

$$P_f[B_1, \mathbf{d}(I_d)] = 1 - P_f[B_1^*, \mathbf{d}(I_d)]$$

$$P_f[B_j, \mathbf{d}(I_d)] = P_f[B_{j-1}^*, \mathbf{d}(I_d)] - P_f[B_j^*, \mathbf{d}(I_d)] \quad (j = 2, 3, 4)$$

$$P_f[B_5, \mathbf{d}(I_d)] = P_f[B_4^*, \mathbf{d}(I_d)]$$



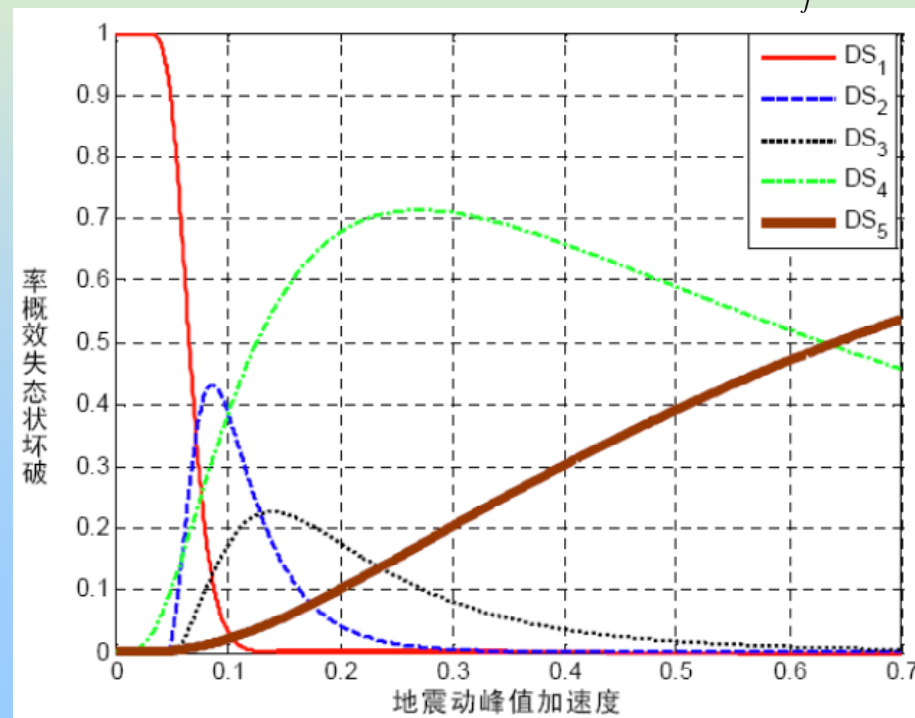
# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Damage Analysis of Structures

### Damage State Fragility Curves

$$P_{DS_j} = P_f[B_j, \mathbf{d}(I_d)] = \int_0^\infty F_{DS_j}(x) d | H_{S_a}(x) |$$

### Damage State Fragility Curves $F_{DS_j}(x)$



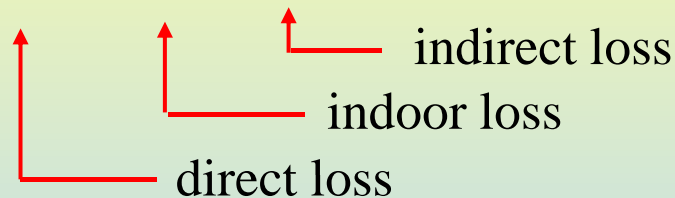
# Probabilistic seismic risk analysis of structures

## □ Probabilistic Seismic Loss Analysis of Structures

### ➤ Earthquake Loss Evaluation of Structures

$$D_j = D_j^{(1)} + D_j^{(2)} + D_j^{(3)} \quad (j = 1, \dots, 5)$$

earthquake loss for five damage states

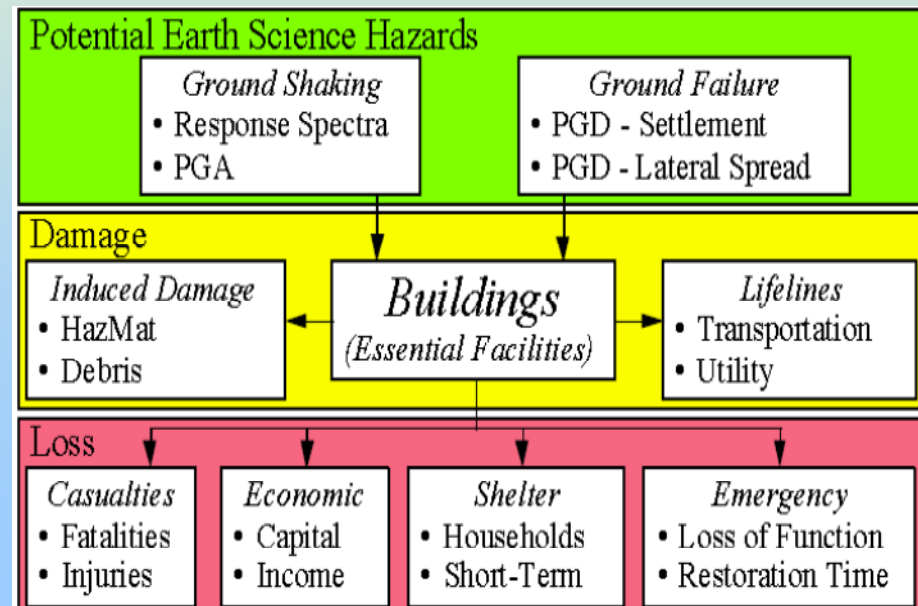


### Loss coefficients method

• direct loss  $D_j^{(1)} = \xi(B_j)C_I(I_d)$

• indoor loss  $D_j^{(2)} = \eta(B_j)C_{eq}$

• indirect loss  $D_j^{(3)} = \gamma(B_j)D_j^{(1)}$





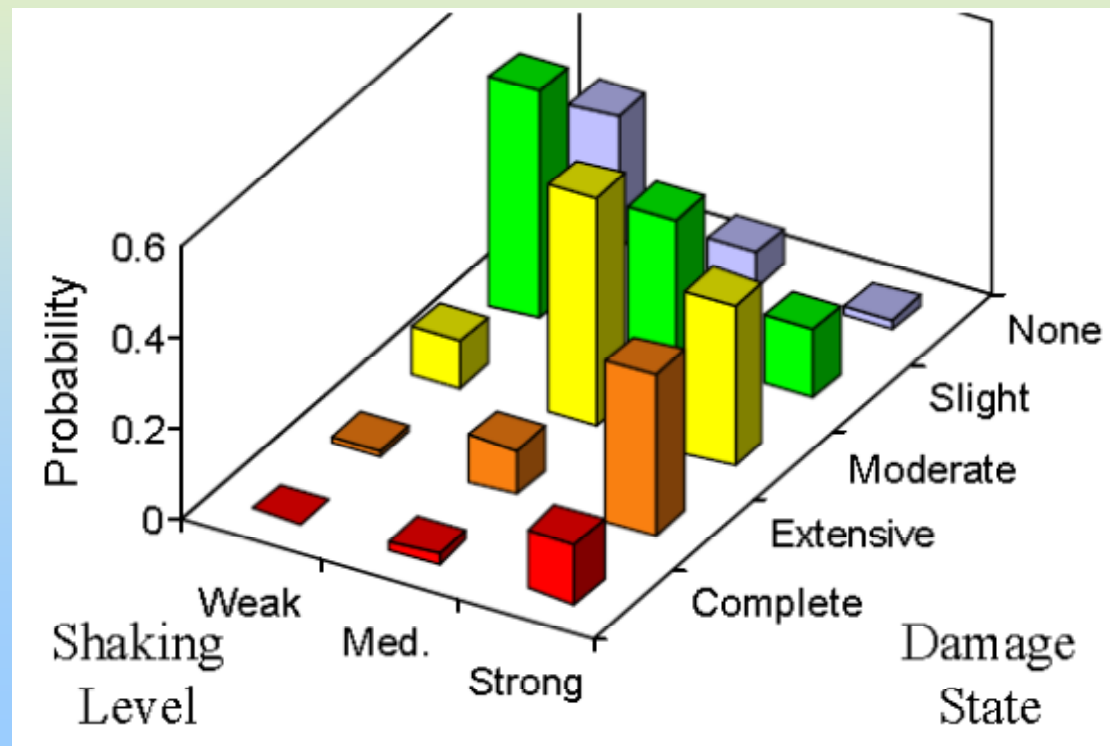
## Minimum life-cycle cost design methodology of aseismic structures

### □ Probabilistic Seismic Loss Analysis of Structures

#### ➤ Expected Failure Cost Analysis

$$L[\mathbf{d}(I_d)] = \sum_{j=1}^5 P_f[B_j, \mathbf{d}(I_d)] \cdot D_j$$

Expected earthquake loss considering five damage states

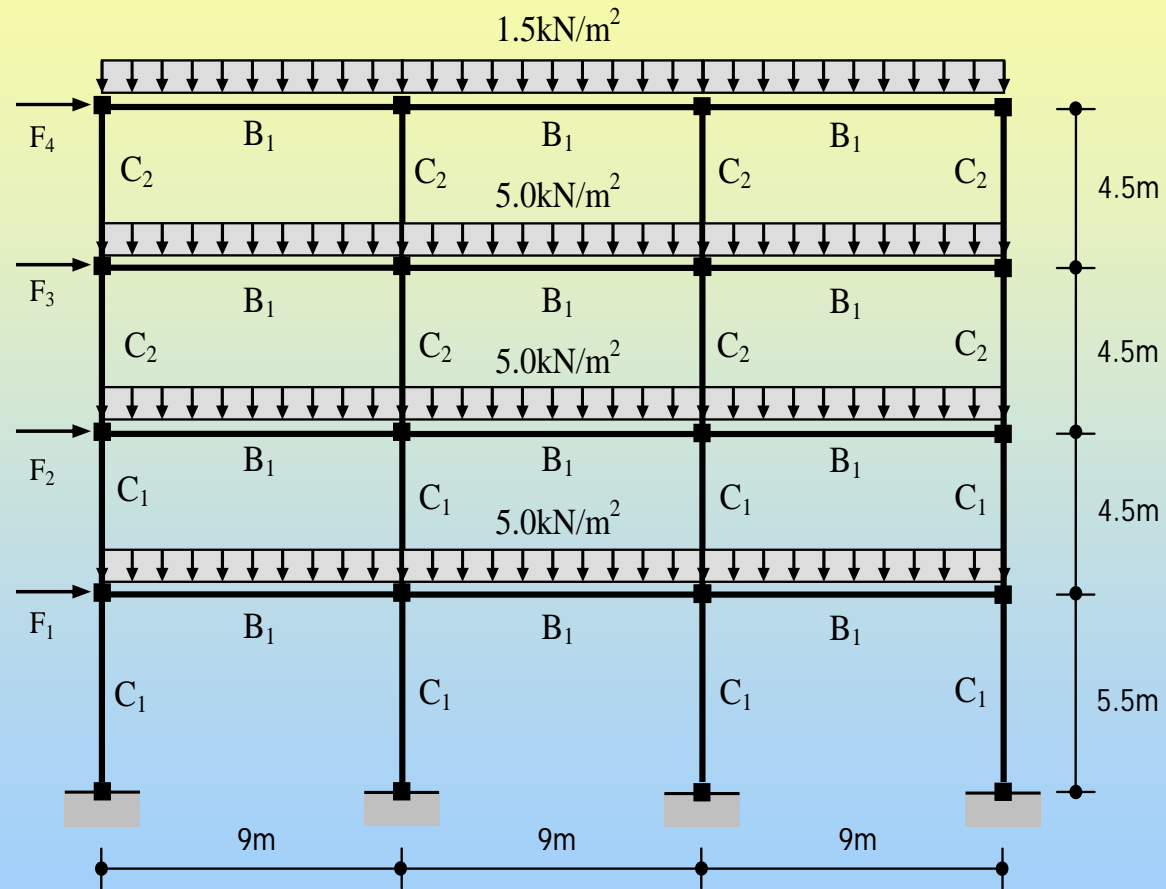


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- ❑ **Conclusions**

# Applications of the methodology to steel frame buildings

## □ Description of the model



Three-bay and four-storey plane steel frame

## Applications of the methodology to steel frame buildings

### □ Loss coefficients for five damage states

Damage State	$\xi$	$\eta$	$\gamma$
<b>B<sub>1</sub>: None</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
<b>B<sub>2</sub>: Slight</b>	<b>0.10</b>	<b>0.05</b>	<b>0.00</b>
<b>B<sub>3</sub>: Moderate</b>	<b>0.30</b>	<b>0.15</b>	<b>0.50</b>
<b>B<sub>4</sub>: Severe</b>	<b>0.90</b>	<b>0.50</b>	<b>2.00</b>
<b>B<sub>5</sub>: Collapse</b>	<b>1.00</b>	<b>0.95</b>	<b>6.00</b>

# Applications of the methodology to steel frame buildings

## □ Results of analysis

### Seismic risk probabilities for five damage states

Intensity $I_d$	$P_f[B_1]$	$P_f[B_2]$	$P_f[B_3]$	$P_f[B_4]$	$P_f[B_5]$
6.0	0.1754	0.3776	0.0001	0.4443	0.0026
6.5	0.2381	0.3203	0.2798	0.1615	0.0003
7.0	0.1250	0.6624	0.1758	0.0364	0.0004
7.5	0.6000	0.3014	0.0710	0.0260	0.0016
8.0	0.7229	0.2417	0.0249	0.0105	0.0000
8.5	0.7254	0.2417	0.0287	0.0042	0.0000
9.0	0.7287	0.2680	0.0033	0.0000	0.0000

# Applications of the methodology to steel frame buildings

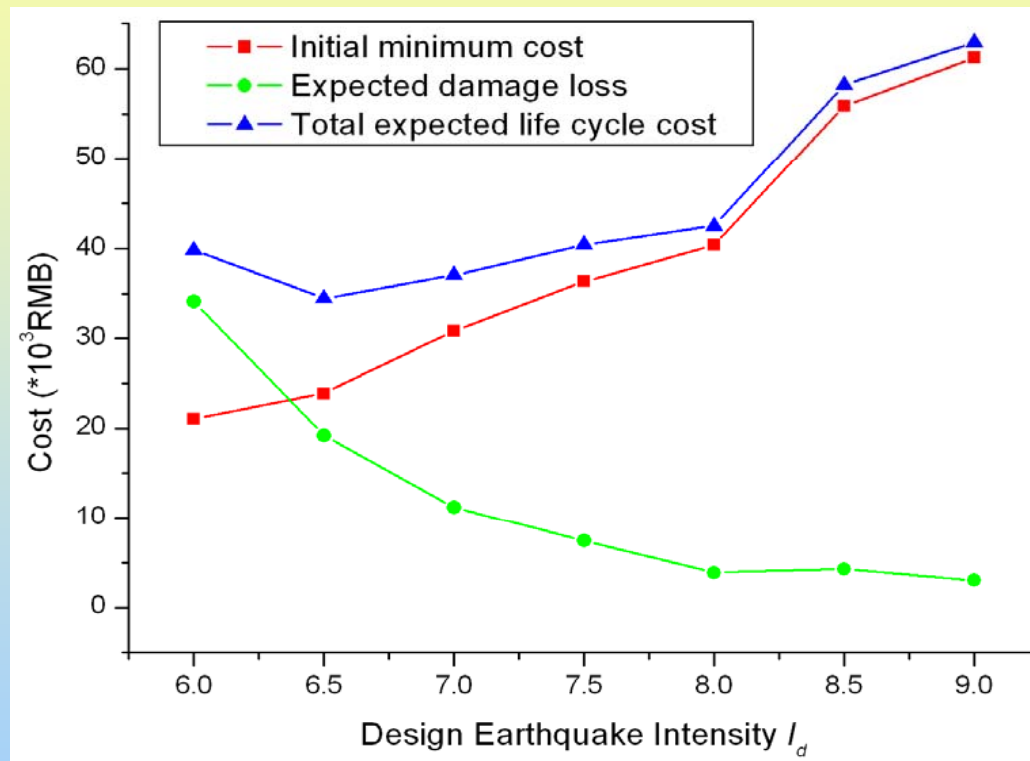
## □ Results of analysis

**Total expected life cycle cost ( $\times 10^5$ RMB)**

Intensity $I_d$	$C_0$	$C_{\min}$	$L$	$E[C_T]$
6.0	27.812	21.032	34.0905	39.8073
6.5	35.213	23.875	19.2103	34.4551
7.0	45.897	30.846	11.2136	37.0219
7.5	73.911	36.336	7.4071	40.4155
8.0	87.296	40.374	3.8489	42.4938
8.5	97.553	55.867	4.2548	58.2103
9.0	115.27	61.254	3.0009	62.9113

# Applications of the methodology to steel frame buildings

## □ Results of analysis



**Decision-making of optimal fortification intensity**

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# Conclusions

- **Rational model of total expected life cycle cost should include the minimum initial cost plus the total expected damage cost.**
- **The optimal fortification intensity represents the acceptable risk level more flexible and convenient than the target reliability**
- **The division of total optimum design process based on life cycle cost into the stage of the decision-making of the optimal fortification load and the stage of minimum initial cost design can greatly overcome some difficulties in the conventional design methods**



**哈爾濱工業大學**  
HARBIN INSTITUTE OF TECHNOLOGY



**Thanks you for your attention!**

**Prof. Dr. Da-Gang LU**  
**School of Civil Engineering**  
**Harbin Institute of Technology**  
**E-mail: [ludagang@hit.edu.cn](mailto:ludagang@hit.edu.cn)**