



哈爾濱工業大學

HARBIN INSTITUTE OF TECHNOLOGY



# 有限元可靠度方法的 基本原理与研究进展

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四川，成都

# 报告提纲

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# 引言

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**MCS**



**(RSM)**

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**SFEM**



**Taylor**

**Neumann**



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” **(Finite Element Reliability Method, FERM)**


# 一次有限元可靠度方法的基本原理

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**S**

**V**


$$\mathbf{S} = \mathbf{S}(\mathbf{V})$$


—


$$g[\mathbf{S}(\mathbf{V}), \mathbf{V}]$$

—

$$p_f = \int_{g[\mathbf{s}(\mathbf{v}), \mathbf{v}] \leq 0} f_{\mathbf{V}}(\mathbf{v}) d\mathbf{v}$$

$$\mathbf{Y} = T(\mathbf{V})$$


$$p_f = \int_{G(\mathbf{y}) \leq 0} \varphi_n(\mathbf{y}) d\mathbf{y}$$


$$G(\mathbf{y}) = g[\mathbf{s}(\mathbf{v}), \mathbf{v}] = g[\mathbf{s}(T^{-1}(\mathbf{y})), \mathbf{y}]$$

## 一次有限元可靠度方法的基本原理

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$$G(\mathbf{y}) \approx G(\mathbf{y}^*) + \nabla_{\mathbf{y}^*} G^T (\mathbf{y} - \mathbf{y}^*) = \nabla_{\mathbf{y}^*} G^T (\mathbf{y} - \mathbf{y}^*)$$

$$p_f \approx p_{f1} = \int_{\beta - \boldsymbol{\alpha}^{*T} \mathbf{y} \leq 0} \varphi_n(\mathbf{y}) d\mathbf{y} = \Phi(-\beta)$$

$$\boldsymbol{\alpha}^* = -\nabla_{\mathbf{y}^*} G / \|\nabla_{\mathbf{y}^*} G\| \quad \beta = \boldsymbol{\alpha}^{*T} \mathbf{y}^*$$

$$\mathbf{y}_{i+1} = \left( \frac{G(\mathbf{y}_i)}{\|\nabla_{\mathbf{y}_i} G\|} + \boldsymbol{\alpha}_i^T \mathbf{y}_i \right) \boldsymbol{\alpha}_i$$

$$\nabla_{\mathbf{y}} G = (\mathbf{J}_{\mathbf{y},\mathbf{v}}^{-1})^T \cdot \nabla_{\mathbf{v}} g = (\mathbf{J}_{\mathbf{y},\mathbf{v}}^{-1})^T \cdot [\nabla_{\mathbf{s}} g \cdot \mathbf{J}_{\mathbf{s},\mathbf{v}} + \nabla_{\mathbf{v}} g]$$

# 线弹性有限元可靠度分析

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$$\mathbf{R} = \bigcup_e \int_{\Omega_e} \mathbf{B}^T \cdot \boldsymbol{\sigma} d\Omega_e = \bigcup_e \left[ \int_{\Omega_e} \mathbf{N}^T \cdot \rho_0 \mathbf{b}_0 d\Omega_e + \int_{\partial\Omega_e} \mathbf{N}^T \cdot \mathbf{t}_0 dS_e \right] = \mathbf{F}$$

$$\mathbf{R} = \mathbf{K}\mathbf{U} = \mathbf{F}$$

—

$$\frac{d\mathbf{R}}{d\mathbf{v}} = \left. \frac{\partial \mathbf{R}}{\partial \mathbf{U}} \right|_{\mathbf{v}} \frac{d\mathbf{U}}{d\mathbf{v}} + \frac{\partial \mathbf{R}}{\partial \mathbf{v}} = \frac{\partial \mathbf{F}}{\partial \mathbf{v}}$$

$$\mathbf{K} \frac{d\mathbf{U}}{d\mathbf{v}} + \frac{\partial \mathbf{R}}{\partial \mathbf{v}} = \frac{\partial \mathbf{F}}{\partial \mathbf{v}}$$

$$\mathbf{J}_{\mathbf{U},\mathbf{v}} = \frac{d\mathbf{U}}{d\mathbf{v}} = \mathbf{K}^{-1} \left( \frac{\partial \mathbf{F}}{\partial \mathbf{v}} - \frac{\partial \mathbf{R}}{\partial \mathbf{v}} \right)$$

# 线弹性有限元可靠度分析

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➤  $\mathbf{V}_p$

➤  $\mathbf{V}_m$

➤  $\mathbf{V}_g$

$$\mathbf{R}[\mathbf{X}(\mathbf{V}_g), \mathbf{U}(\mathbf{X}(\mathbf{V}_g), \mathbf{V}_m, \mathbf{V}_p), \mathbf{V}_m] = \mathbf{F}[\mathbf{X}(\mathbf{V}_g), \mathbf{V}_p]$$

—

$$\mathbf{J}_{\mathbf{U}, \mathbf{V}_p} = \frac{d\mathbf{U}}{d\mathbf{V}_p} = \mathbf{K}^{-1} \left( \frac{\partial \mathbf{F}}{\partial \mathbf{V}_p} \right)$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{V}_p} = \bigcup_e \left[ \int_{\Omega_e} \mathbf{N}^T \cdot \frac{\partial(\rho_0 \mathbf{b}_0)}{\partial \mathbf{V}_p} d\Omega_e + \int_{\partial\Omega_e} \mathbf{N}^T \cdot \frac{\partial \mathbf{t}_0}{\partial \mathbf{V}_p} dS_e \right]$$

# 线弹性有限元可靠度分析

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$$\mathbf{J}_{\mathbf{U}, \mathbf{v}_m} = \frac{d\mathbf{U}}{d\mathbf{v}_m} = \mathbf{K}^{-1} \left( -\frac{\partial \mathbf{R}}{\partial \mathbf{v}_m} \right)$$

$$\frac{\partial \mathbf{R}}{\partial \mathbf{v}_m} = \bigcup_e \int_{\Omega_e} \mathbf{B}^T \cdot \frac{\partial \mathbf{D}}{\partial \mathbf{v}_m} \cdot \mathbf{B} \cdot \mathbf{U}_e d\Omega_e$$

—

$$\mathbf{J}_{\mathbf{U}, \mathbf{v}_g} = \frac{d\mathbf{U}}{d\mathbf{v}_g} = \mathbf{K}^{-1} \left( \frac{\partial \mathbf{F}}{\partial \mathbf{X}} - \frac{\partial \mathbf{R}}{\partial \mathbf{X}} \right) \cdot \frac{\partial \mathbf{X}}{\partial \mathbf{v}_g}$$



# 线弹性有限元可靠度分析

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$$\mathbf{J}_{\varepsilon, \mathbf{v}} = \mathbf{B}\mathbf{J}_{\mathbf{U}, \mathbf{v}}$$

—

$$\mathbf{J}_{\sigma, \mathbf{v}} = \mathbf{D}\mathbf{B}\mathbf{J}_{\mathbf{U}, \mathbf{v}} + \mathbf{J}_{\mathbf{D}, \mathbf{v}}\mathbf{B}\mathbf{U}$$

—

$$\nabla_{\mathbf{y}} G = (\mathbf{J}_{\mathbf{y}, \mathbf{v}}^{-1})^T \cdot \nabla_{\mathbf{v}} g = (\mathbf{J}_{\mathbf{y}, \mathbf{v}}^{-1})^T \cdot [\nabla_{\mathbf{s}} g \cdot \mathbf{J}_{\mathbf{s}, \mathbf{v}} + \nabla_{\mathbf{v}} g]$$

$$\boldsymbol{\lambda}^T \mathbf{K} = \nabla_{\mathbf{s}} g$$

$$\nabla_{\mathbf{U}} g \cdot \mathbf{J}_{\mathbf{U}, \mathbf{v}} = \boldsymbol{\lambda}^T \left( \frac{\partial \mathbf{F}}{\partial \mathbf{v}} - \frac{\partial \mathbf{R}}{\partial \mathbf{U}} \Big|_{\mathbf{U}} \right)$$

# 几何非线性有限元可靠度分析

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$$\mathbf{R} = \bigcup_e \int_{\Omega_e} (\mathbf{B}_L + \mathbf{B}_N)^T \sigma_S d\Omega_e$$

$$\mathbf{F} = \bigcup_e \left[ \int_{\Omega_e} \mathbf{N}^T \cdot \rho_0 \mathbf{b}_0 d\Omega_e + \int_{\partial\Omega_e} \mathbf{N}^T \cdot \mathbf{t}_0 dS_e \right]$$

$$\sigma_S = E \varepsilon_G = E (\mathbf{B}_L + \mathbf{B}_N)^T \mathbf{U}$$

$$\mathbf{F} = \mathbf{F}[\mathbf{X}(\mathbf{V}_g), \mathbf{U}(\mathbf{X}(\mathbf{V}_g), \mathbf{V}_m, \mathbf{V}_p), \mathbf{V}_p]$$

—

$$\mathbf{J}_{\mathbf{U}, \mathbf{v}} = \frac{d\mathbf{U}}{d\mathbf{v}} = \mathbf{K}_T^{-1} \left( \frac{\partial \mathbf{F}}{\partial \mathbf{v}} - \frac{\partial \mathbf{R}}{\partial \mathbf{v}} \right)$$

$$\mathbf{K}_T = \frac{\partial \mathbf{R}}{\partial \mathbf{U}} = \bigcup_e \int_{\Omega_e} \left[ \frac{\partial \mathbf{B}_N^T}{\partial \mathbf{U}} \sigma_S + (\mathbf{B}_L + \mathbf{B}_N)^T \cdot \frac{\partial \sigma_S}{\partial \mathbf{U}} \right] d\Omega_e$$

# 几何非线性有限元可靠度分析

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$$\mathbf{K}_T = \mathbf{K}_E + \mathbf{K}_G + \mathbf{K}_U$$

$$\mathbf{K}_E = \bigcup_e \int_{\Omega_e} E \mathbf{B}_L^T \mathbf{B}_L^T d\Omega_e$$

$$\mathbf{K}_G = \bigcup_e \int_{\Omega_e} \frac{\partial \mathbf{B}_N^T}{\partial \mathbf{U}} \sigma_s d\Omega_e$$

$$\mathbf{K}_U = \bigcup_e \int_{\Omega_e} E \left[ \mathbf{B}_L^T \mathbf{B}_N^T + \mathbf{B}_N^T \mathbf{B}_L^T + \mathbf{B}_N^T \mathbf{B}_N^T \right] d\Omega_e$$

# 材料非线性有限元可靠度分析

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$$\Delta \mathbf{R} = \Delta \mathbf{F} + \Delta \mathbf{S}_p$$

$$\Delta \mathbf{R} = \mathbf{K}_0 \Delta \mathbf{U}$$

$$\Delta \mathbf{F} = \bigcup_e \left[ \int_{\Omega_e} \mathbf{N}^T \cdot \rho_0 \Delta \mathbf{b}_0 d\Omega_e + \int_{\partial\Omega_e} \mathbf{N}^T \cdot \Delta \mathbf{t}_0 dS_e \right]$$

$$\Delta \mathbf{S}_p = \mathbf{K}_p \Delta \mathbf{U}$$

$$\mathbf{K}_0 = \bigcup_e \left( \int_{\partial\Omega_e} \mathbf{B}^T \cdot \mathbf{E}_{el} \cdot \mathbf{B} d\Omega_e \right)$$

$$\mathbf{K}_p = \bigcup_e \left( \int_{\partial\Omega_e} \mathbf{B}^T \cdot \mathbf{E}_{pl} \cdot \mathbf{B} d\Omega_e \right)$$

# 材料非线性有限元可靠度分析

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$$\mathbf{J}_{\mathbf{U},\mathbf{v}} = \frac{d\Delta\mathbf{U}}{d\mathbf{v}} = \mathbf{K}_T^{-1} \left( \frac{\partial\Delta\mathbf{F}}{\partial\mathbf{v}} + \frac{\partial\Delta\mathbf{S}_p}{\partial\mathbf{v}} - \frac{\partial\mathbf{R}}{\partial\mathbf{v}} \right)$$

$$\mathbf{K}_T = \mathbf{K}_0 - \mathbf{K}_p$$

## 系統有限元可靠度分析

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$$\mathbf{R} = (\mathbf{K} + \mathbf{K}_R)\mathbf{U} + \mathbf{R}_R = \mathbf{F}$$

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$$\frac{d\mathbf{R}}{d\mathbf{v}} = \frac{\partial \mathbf{R}}{\partial \mathbf{U}} \Big|_{\mathbf{v}} \frac{d\mathbf{U}}{d\mathbf{v}} + \frac{\partial \mathbf{R}}{\partial \mathbf{v}} \Big|_{\mathbf{U}} + \frac{d\mathbf{R}_R}{d\mathbf{v}} = (\mathbf{K} + \mathbf{K}_R) \frac{d\mathbf{U}}{d\mathbf{v}} + \frac{\partial \mathbf{R}}{\partial \mathbf{v}} \Big|_{\mathbf{U}} + \frac{d\mathbf{R}_R}{d\mathbf{v}} = \frac{d\mathbf{F}}{d\mathbf{v}}$$

$$\mathbf{J}_{\mathbf{U},\mathbf{v}} = \frac{d\Delta\mathbf{U}}{d\mathbf{v}} = (\mathbf{K} + \mathbf{K}_R)^{-1} \left( \frac{d\mathbf{F}}{d\mathbf{v}} - \frac{\partial \mathbf{R}}{\partial \mathbf{v}} \Big|_{\mathbf{U}} - \frac{d\mathbf{R}_R}{d\mathbf{v}} \right)$$

## 时变有限元可靠度分析

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$$\mathbf{M}(\theta)\ddot{\mathbf{U}}(t, \theta) + \mathbf{C}(\theta)\dot{\mathbf{U}}(t, \theta) + \mathbf{R}[\mathbf{U}(t, \theta), \theta] = \mathbf{F}(t, \theta)$$

—

**Newmark— $\beta$**

$$\ddot{\mathbf{U}}_{n+1} = \left(1 - \frac{1}{2\beta}\right)\ddot{\mathbf{U}}_n - \frac{1}{\beta(\Delta t)}\dot{\mathbf{U}}_n + \frac{1}{\beta(\Delta t)^2}(\mathbf{U}_{n+1} - \mathbf{U}_n)$$

$$\dot{\mathbf{U}}_{n+1} = \Delta t \left(1 - \frac{\alpha}{2\beta}\right)\ddot{\mathbf{U}}_n + \left(1 - \frac{\alpha}{\beta}\right)\dot{\mathbf{U}}_n + \frac{\alpha}{\beta(\Delta t)}(\mathbf{U}_{n+1} - \mathbf{U}_n)$$

## 时变有限元可靠度分析

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$$\Psi(\mathbf{U}_{n+1}) = \tilde{\mathbf{F}}_{n+1} - \left[ \frac{1}{\beta(\Delta t)^2} \mathbf{M}\mathbf{U}_{n+1} + \frac{\alpha}{\beta(\Delta t)} \mathbf{C}\mathbf{U}_{n+1} + \mathbf{R}(\mathbf{U}_{n+1}) \right] = \mathbf{0}$$

$$\begin{aligned} \tilde{\mathbf{F}}_{n+1} = & \mathbf{F}_{n+1} + \mathbf{M} \left[ \frac{1}{\beta(\Delta t)^2} \mathbf{U}_n + \frac{1}{\beta(\Delta t)} \mathbf{C}\dot{\mathbf{U}}_n - \left( 1 - \frac{1}{2\beta} \right) \ddot{\mathbf{U}}_n \right] \\ & + \mathbf{C} \left[ \frac{\alpha}{\beta(\Delta t)^2} \mathbf{U}_n + \left( 1 - \frac{\alpha}{\beta} \right) \dot{\mathbf{U}}_n - (\Delta t) \left( 1 - \frac{\alpha}{2\beta} \right) \ddot{\mathbf{U}}_n \right] \end{aligned}$$

$$\mathbf{R}(\mathbf{U}_{n+1}) = \bigcup_e \left( \int_{\partial\Omega_e} \mathbf{B}^T \cdot \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_{n+1}) d\Omega_e \right)$$



# 时变有限元可靠度分析

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$$(\mathbf{K}_T^{\text{dyn}})^i_{n+1} \Delta \mathbf{U}_{n+1}^i = \Psi(\mathbf{U}_{n+1}^i) \quad i = 0, 1, 2, \dots$$

$$\Psi_{n+1}^i = \tilde{\mathbf{F}}_{n+1} - \left[ \frac{1}{\beta(\Delta t)^2} \mathbf{M} \mathbf{U}_{n+1}^i + \frac{\alpha}{\beta(\Delta t)} \mathbf{C} \mathbf{U}_{n+1}^i + \mathbf{R}(\mathbf{U}_{n+1}^i) \right]$$

—

$$(\mathbf{K}_T^{\text{dyn}})^i_{n+1} = \left[ \frac{1}{\beta(\nabla t)^2} \mathbf{M} + \frac{1}{\beta(\nabla t)} \mathbf{C} + (\mathbf{K}_T^{\text{stat}})^i_{n+1} \right]$$

$$(\mathbf{K}_T^{\text{stat}})^i_{n+1} = \bigcup_e \left( \int_{\partial\Omega_e} \mathbf{B}^T \cdot \mathbf{D}_T \cdot \mathbf{B} d\Omega_e \right)$$

# 时变有限元可靠度分析

$$\frac{d\mathbf{U}_{n+1}}{d\theta} = \left[ (\mathbf{K}_T^{\text{dyn}})_{n+1}^i \right]^{-1} \left[ \frac{\partial \tilde{\mathbf{F}}_{n+1}}{\partial \theta} - \frac{\partial \mathbf{R}(\mathbf{U}_{n+1}(\theta), \theta)}{\partial \theta} \Big|_{\mathbf{U}_{n+1}} - \left( \frac{1}{\beta(\Delta t)^2} \frac{\partial \mathbf{M}}{\partial \theta} + \frac{\alpha}{\beta(\Delta t)} \frac{\partial \mathbf{C}}{\partial \theta} \right) \mathbf{U}_{n+1} \right]$$

$$\begin{aligned} \frac{\partial \tilde{\mathbf{F}}_{n+1}}{\partial \theta} &= \frac{\partial \mathbf{F}_{n+1}}{\partial \theta} + \frac{\partial \mathbf{M}}{\partial \theta} \left[ \frac{1}{\beta(\Delta t)^2} \mathbf{U}_n + \frac{1}{\beta(\Delta t)} \dot{\mathbf{U}}_n - \left( 1 - \frac{1}{2\beta} \right) \ddot{\mathbf{U}}_n \right] \\ &+ \mathbf{M} \left[ \frac{1}{\beta(\Delta t)^2} \frac{\partial \mathbf{U}_n}{\partial \theta} + \frac{1}{\beta(\Delta t)} \frac{\partial \dot{\mathbf{U}}_n}{\partial \theta} - \left( 1 - \frac{1}{2\beta} \right) \frac{\partial \ddot{\mathbf{U}}_n}{\partial \theta} \right] \\ &+ \frac{\partial \mathbf{C}}{\partial \theta} \left[ \frac{\alpha}{\beta(\Delta t)} \mathbf{U}_n - \left( 1 - \frac{\alpha}{\beta} \right) \dot{\mathbf{U}}_n - (\Delta t) \left( 1 - \frac{\alpha}{2\beta} \right) \ddot{\mathbf{U}}_n \right] \\ &+ \mathbf{C} \left[ \frac{\alpha}{\beta(\Delta t)} \frac{\partial \mathbf{U}_n}{\partial \theta} - \left( 1 - \frac{\alpha}{\beta} \right) \frac{\partial \dot{\mathbf{U}}_n}{\partial \theta} - (\Delta t) \left( 1 - \frac{\alpha}{2\beta} \right) \frac{\partial \ddot{\mathbf{U}}_n}{\partial \theta} \right] \end{aligned}$$

$$\frac{\partial \mathbf{R}(\mathbf{U}_{n+1}(\theta), \theta)}{\partial \theta} \Big|_{\mathbf{U}_{n+1}} = \bigcup_e \left( \int_{\partial\Omega_e} \mathbf{B}^T \cdot \frac{\partial \boldsymbol{\sigma}}{\partial \theta} \Big|_{\boldsymbol{\varepsilon}_{n+1}} d\Omega_e \right)$$

# 时变有限元可靠度分析

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— **FORM**

$$w(\mathbf{v}, t) = r(\mathbf{v}, t) - s(\mathbf{v}, t) \quad \text{—}$$

$$\tilde{g}(\mathbf{v}) = \min_{t \in [a, b]} w(\mathbf{v}, t) \quad \text{—}$$

$$g(\mathbf{v}) = \min_{n \in \mathfrak{N}} w_n(\mathbf{v}) \quad \text{—}$$

$$w_n(\mathbf{v}) = w(\mathbf{v}, t_n)$$

$$\mathfrak{N} = \{0, 1, \dots, N\}$$

$$g(\mathbf{v}) = w_{\hat{n}}(\mathbf{v}) \quad \text{—}$$

$$\hat{n} \quad \text{—}$$

# 时变有限元可靠度分析

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## — FORM

$$\mathbf{d}_k = -f(\mathbf{v}_k) + \frac{(\nabla f(\mathbf{v}_k), \nabla w_{\hat{n}_k}(\mathbf{v}_k))}{\|\nabla w_{\hat{n}_k}(\mathbf{v}_k)\|^2} \nabla w_{\hat{n}_k}(\mathbf{v}_k)$$

—

$$f(t) = \mu(t) + \sum_{i=1}^n y_i b_i(t) = \mu(t) + \mathbf{b}(t)^T \mathbf{y}$$



➤ **Karhunen-Loeve**



# 空变有限元可靠度分析

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$$z(\mathbf{x}) = \mu(\mathbf{x}) + \mathbf{b}(\mathbf{x})^T \mathbf{y}$$



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• **Karhunen-Loeve**

## 空变有限元可靠度分析

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$$p_{\mathcal{D}}(\mathbf{x}, t) = P[\mathbf{s}(\mathbf{y}, \mathbf{x}, t) \in \mathcal{D}(\mathbf{x}, t)]$$

—      —

$$p_{\mathcal{D}} = P[\mathbf{s}(\mathbf{y}, \mathbf{x}, t) \in \mathcal{D}(\mathbf{x}, t) \mid \mathbf{x} \in \Omega, t \in T]$$

—      —

$$p_{\mathcal{D}}(t) = P[\mathbf{s}(\mathbf{y}, \mathbf{x}, t) \in \mathcal{D}(\mathbf{x}) \mid \mathbf{x} \in \Omega]$$

—

$$p_{\mathcal{D}}(t) > P\left[\bigcup_i \{\mathbf{s}(\mathbf{y}, \mathbf{x}_i, t) \in \mathcal{D}(\mathbf{x}_i) \mid\}\right]$$

## 空变有限元可靠度分析

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### FORM

$$\begin{aligned} p_{\mathcal{D}}(t) &= P[\mathbf{s}(\mathbf{y}, \mathbf{x}, t) \in \mathcal{D}(\mathbf{x}) \mid \mathbf{x} \in \Omega] \\ &= P[\{\mathbf{s}(\mathbf{y}, \mathbf{x}_0, t) \in \mathcal{D}(\mathbf{x}_0)\} \cup \{0 < N_{\mathcal{D}}(\Omega)\}] \\ &\leq p_{\mathcal{D}}(\mathbf{x}_0, t) + P[0 < N_{\mathcal{D}}(\Omega)] \\ &= p_{\mathcal{D}}(\mathbf{x}, t) + \int_{\Omega} v_{\mathcal{D}}(\mathbf{x}, t) d\mathbf{x} \end{aligned}$$

$$v_{\mathcal{D}}(x, t) = \lim_{\delta x \rightarrow 0} \frac{P[\{\mathbf{s}(\mathbf{y}, x, t) \notin \mathcal{D}(x)\} \cap \{\mathbf{s}(\mathbf{y}, x + \delta x, t) \notin \mathcal{D}(x + \delta x)\}]}{\delta x}$$

# 有限元可靠度软件

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## – STRUREL

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RCP



COMREL



SYSREL



STATREL



Permas-RA

## – Slang

ISM

## – NESSUS

NASA



# 有限元可靠度软件

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– **ISPUD COSSAN**

**(IFM)**

**G.I. Schuëller**

– **COMPASS**

**Martec**

– **ProSINTAP**

**Failure Assessment Diagram, FDA**

**MCS**

**FORM**

# 有限元可靠度软件

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## – CALREL-FEAP

Der

Kiureghian

20 80

## – SYSREL-FEAP

Frangopol

2000

# 有限元可靠度软件

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## – FERUM

Kiureghian	Haukaas	Song	Der
Matlab			2000

## – OpenSees-Reliability

Kiureghian	Haukaas	Der
(PEER)		
OpenSees		

# 有限元可靠度软件

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– FERAP

( 2005)

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Unified Modeling Language, UML

FERAP(Finite Element Reliability Program  
FORM

# 有限元可靠度方法的工程应用

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➤ (A. Der Kiureghian & Ke)

➤ (A. Haldar & S. Mahadevan)

➤ (D. Frangopol)

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FORM

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FORM

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# 有限元可靠度方法的工程应用

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□ D. Frangopol & K. Imai



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□ 1996, 1997, 1999, 2001, 2004

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□ 2001

# 结 论

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谢谢！

Thank you for your  
attention !